Competition in the health care market: a "two-sided" approach

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Abstract

Two identical hospitals compete for patients and doctors choosing locations (e.g. specializations) on a Hotelling line, and selecting the quality of the treatment and the salary for the doctors. Patients pay the price chosen by a benevolent central planner. Introducing the presence of cross-group externalities for the patients (i.e. ceteris paribus patients prefer the hospital with the highest number of doctors), we show that in equilibrium hospitals always maximally differentiate their services. The regulator, choosing the price, can affect only the provision of quality that, in equilibrium, may be provided at the socially optimal level.

Keywords: price regulation, quality competition, spatial competition, gatekeeping

JEL classification: L13, L50, R30, R38, D82, I11, I18

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1. Introduction

The paper studies the competition between two identical hospitals that compete for patients and doctors choosing specializations, quality and salaries, when prices are regulated by a benevolent central planner. In particular, patients and doctors are uniformly distributed on a segment of unitary length and incur quadratic mismatch costs to access a hospital located at a positive distance. Assuming in addition that patients experience appositive cross-group externality with respect to the number of doctors employed in a hospital, our model connects two recent and growing branches of literature: contributions regarding imperfect hospital non price competition, and works concerning the study of competition in “two-sided” markets. On one side, recent contributions of health economics study the strategic behaviour of hospitals that compete for patients choosing the specialization and the quality of their service when prices are in general regulated by a benevolent central planner. In particular, the basic frameworks of the linear (H. Hotelling, 1929) city and the circular (S.C. Salop, 1979) city have been used, following the extension proposed by (N. Economides, 1993, 1989) in which a quality choice stage is introduced in the game. Examples of papers studying (duopolistic) hospital competition when locations are on the line are given by (P.S. Calem and J.A. Rizzo, 1995), (K. Brekke et al., 2006), (K. Brekke et al., 2007). While in the first two papers agents are assumed to be perfectly informed, in the last one the authors introduce the possibility that patients are not informed about the disease they suffer and about the quality and specialization choices of the hospitals. The common result of these papers is that in equilibrium the level of product differentiation and the provision of quality can be respectively excessive (insufficient) and insufficient (excessive) compared to the social optimum. In (P.S. Calem and J.A. Rizzo,

1995) it is assumed that hospitals share part of the transportation costs (mismatch costs) that patients incur to reach the chosen hospital; prices are regulated, but the analysis of optimal regulation is missing. (K. Brekke, R. Nuscheler and O.R. Straume, 2006) study optimal price regulation adopting a similar framework, removing, however, the assumption that hospitals share mismatch costs with their patients. The Hotelling framework is borrowed also in (P.P. Barros and X. Martinez-Giralt, 2002); however, the authors are interested in studying the competition (in prices and qualities) between two hospitals when a third-party payer bears part of the patient's treatment price depending on different co-payment systems. Papers that study oligopolistic competition in the health market using the framework of Salop's circular city are (H. Gravelle, 1999), (H. Gravelle and G. Masiero, 2000), and (R. Nuscheler, 2003). Assuming hospitals (or in particular general practitioners, GPs, for the first two papers) located on the circumference of a circle, the interest of the authors if focused to the entry decisions rather than the specialization choice of health services.

What in our opinion has not been fully explored in this literature is the role played by the input side of the market, i.e. the doctors. Hospitals compete on one side for patients, but at the same time they compete to attract doctors. Some decision variables can be specific, such as salaries or productivity bonuses; others variables (such as quality and specialization) chosen by the hospitals affect the decisions of doctors as well as patients. Doctors could present different preferences towards the specialization chosen by the hospitals; in addition, in our opinion it is reasonable to assume that ceteris paribus all doctors would receive a higher utility if hired by the highest quality hospital.

Modelling hospitals' competition for doctors introduces another aspect that the literature mentioned above has not studied yet: indirect network externalities. Such a feature has been studied in the literature concerning the so called *two-sided* markets ((B. Caillaud and B. Jullien, 2003), (M. Armstrong, 2005, M. Armstrong and J. Wright, 2004), (J-C. Rochet and J. Tirole, 2003)). In a two-sided market agents belonging to two different groups receive utility only from interacting with the members of the other group through the service of a platform. The number of agents of one group that join a platform generates an indirect network externality to the member of the other group joining the same platform. Typical examples of such markets are the broadcasting industry, the payment cards sector, software industry. In our opinion, hospitals can be thought as platforms facing patients and doctors on the two sides of the market for health care, especially for services that can be offered only with the facilities provided in a hospital. (D. Bardey and J-C. Rochet, 2006) introduced such an idea studying the competition of health plans (instead of hospitals) for patients and doctors. In their model patients have different probabilities of being ill and they benefit from belonging to a health plan with a large number of doctors, who in turn are distributed on a Hotelling line and are paid on a fee-for-service regime (implying that doctors too have a positive cross-group externality towards the number of policy holders). Hospital plans compete in the level of doctors' remuneration and the premium. The authors show that less restrictive plans in equilibrium may obtain higher profits due to the presence of cross-group externalities.

The aim of our paper is instead to study hospital (duopolistic) competition when both patients and doctors are distributed on a line and hospitals compete for both groups choosing qualities, salaries and locations. We shall assume that prices are regulated by a benevolent central planner. We want to study the case where ceteris paribus patients receive higher utility from being served in a bigger (in terms of number of doctors) hospital, while doctors (being their salary not related to the number of patients served) do not have any externality regarding the number of patients who decide to be served by a hospital. Initially, we will study a model of complete information (as in (K. Brekke, R. Nuscheler and O.R. Straume, 2006)), then we will study the case in which patients are imperfectly informed about hospitals' strategic decisions and about the disease they suffer, and GPs would play a gatekeeping role in the system, (as in (K. Brekke, R. Nuscheler and O.R. Straume, 2007)).

The timing of the game (similar to (K. Brekke, R. Nuscheler and O.R. Straume, 2007)) is given as follows:

- In stage 1 the regulator sets prices and decides whether all patients should visit a GP (at a cost) in order to acquire the necessary information regarding the hospitals (strict gatekeeping).
- In stage 2 hospitals simultaneously choose specialisations.
- In stage 3 hospitals simultaneously choose qualities and salaries.
- In stage 4 patients and doctors choose a hospital. If the regulator can not impose a regime of strict gatekeeping, an additional stage can be introduced in between stage 3 and 4, in which patients choose whether to visit a GP at a cost.

We show that, explicitly modelling the doctors' side of the market, competition increases to the point that for any combination of parameters hospitals will choose to maximally differentiate their services. In contrast to what has been shown by (K. Brekke, R. Nuscheler and O.R. Straume, 2006) under the assumption of complete information and by (K. Brekke,

R. Nuscheler and O.R. Straume, 2007) under the assumption that only a portion of the patients are informed regarding their illness and hospitals' characteristics, the regulator can now only indirectly affect the quality provided through the price selection and, indeed, achieve in equilibrium the socially optimal quality provision. Location choice, instead, is not affect by the price chosen at the beginning of the game and being maximal differentiation suboptimal, the equilibrium can only reach a second best situation.

The rest of the paper is structured as follows. Section two studies the model with complete information and describes the results. Section three introduces incomplete information and a gatekeeping role for GPs. Section four concludes.

2. The model with informed patients

Let us the market for hospital services be described by four groups of agents.

Patients

Patients demand inelastically one unit of hospital care. They are distributed uniformly with density equal to one on a segment of length equal to one. To access a hospital they need to pay the regulated price $p > 0$. In addition, patients incur quadratic mismatch cost when they are treated in a hospital not specialized in their type of illness. The indirect utility, net of the mismatch and access costs, of the generic patients located at location *z* treated in hospital *i* located at x_i, *i=1,2* is given by:

$$u_i^p = v^p + \gamma n_i^d + q_i - t_p \left(z - x_i \right)^2 - p \tag{1}$$

Where $v^p > 0$ is the utility that each patient obtains when recovered from her disease, $n_i^d \in [0,1]$ is the number of doctors working for hospital *i*, $\gamma \geq 0$ is the measure of the externality experienced by the patients when an extra doctor works for hospital *i*; $q_i > 0$ is the quality chosen by hospital *i*; $t_p > 0$ is the mismatch cost parameter.

Doctors

Every doctor supplies inelastically one unit of labour. They are uniformly distributed with density equal to one on a segment of length equal to one. Working in a hospital is their only source of utility. The indirect utility of the generic doctor located at *y* employed by hospital *i* located at x_i, *i=1,2,* is given by:

$$u_i^d = v^d + \vartheta q_i + w_i - t_d \left(y - x_i \right)^2 \quad (2)$$

where $v^d > 0$ is the utility that doctors obtain from their job, $\vartheta \geq 0$ is the measure of the benefit that doctors obtain from working in a hospital providing quality equal to 1; $t_d > 0$ is the parameter of the mismatch costs doctors incur from working in a hospital not specialized i their preferred treatment and $w_i \geq 0$ is the wage paid by hospital *i*.

Hospitals

There are two hospitals, indexed by *i=1,2.* They choose their locations[1] (specialization) on a segment of length equal to one; in particular, let us define x_i, *i=1,2,* the location of hospital *i*, and let us assume without loss of generality that $x_1 \leq x_2$ and $\Delta = x_2 - x_1$. Hospitals can also invest in quality in order to attract patients and doctors. We assume

[1] Symmetric by assumption.

fixed costs for quality enhancement. In order to attract doctors, hospitals choose also the salary $w_i \geq 0$.

The profit function for the generic hospital *i* is therefore given by:

$$\Pi_i = p\, n_i^p - w_i\, n_i^d - k\frac{q_i^2}{2} \qquad (3)$$

where $n_i^p \in [0,1]$ is the number of patients served by hospital *i*, $kq_i^2/2$ are the fixed costs of quality enhancement and $k > 0$ is the parameter measuring the intensity of costs for quality.

Regulator

The regulator is a benevolent central planner who sets the price of the hospital services in order to maximize social welfare (given by the summation of patients, doctors and hospitals' surplus), subject to the constraint that hospitals do not earn negative profits. We are therefore focusing in the study of a prospective payment system such as the DRG-pricing system.

We want to study the Subgame Perfect Nash Equilibrium, SPNE, of a three stage game[2] in which, in stage 1 the regulator sets the price $p > 0$, in stage 2 hospitals choose specializations x_i, *i=1,2,* and in stage 3 hospitals compete for patients and doctors maximizing the profit functions choosing qualities and wages. We solve the game by the method of backward induction.

[2] This is a particular version (the regulator does not need to impose a strict gatekeeping regime since patients are assumed to be informed) of the sequencing we described in the previous section.

Let us start from stage 3 of the game. For given price and specializations, hospitals choose simultaneously and not cooperatively quality and wages in order to attract patients and doctors and maximize profits. The only source of revenues comes from the price paid by patients, however hospitals compete for doctors since they generate a positive network externality on patients. In order to study hospitals' payoff functions, we need to define n_i^k, $k = d, p$, i.e. the demand of patients and the supply of doctors for each hospital. To do so, we look for the marginal patient and the marginal doctor, i.e. the patient and the doctor who are indifferent to access either of the two hospitals. The marginal patient is located at $\bar{z} \in [0,1]$ given by:

$$u_1^p(\bar{z}) = u_2^p(\bar{z}) \Rightarrow \bar{z} = \frac{1}{2} + \frac{1}{2t_p\Delta}\left[\gamma\left(n_1^d - n_2^d\right) + \left(q_1 - q_2\right)\right] \tag{4}$$

and the marginal doctor is located at $\bar{y} \in [0,1]$ given by:

$$u_1^d(\bar{y}) = u_2^d(\bar{y}) \Rightarrow \bar{y} = \frac{1}{2} + \frac{1}{2t_d\Delta}\left[\vartheta\left(q_1 - q_2\right) + \left(w_1 - w_2\right)\right] \tag{5}$$

Given the assumptions regarding patients and doctors' behaviour and distribution we know that:

$$\begin{aligned} n_1^p &= \bar{z} \qquad & n_2^p &= 1 - n_1^p \\ n_1^d &= \bar{y} \qquad & n_2^d &= 1 - n_1^d \end{aligned} \tag{6}$$

Substituting (4) and (5) into (6), and solving the equalities expressed in (6), we obtain:

$$\begin{aligned} n_1^p &= \frac{\gamma\left(w_1 - w_2 + \vartheta\left(q_1 - q_2\right)\right) + t_d\Delta\left(q_1 - q_2 + t_p\Delta\right)}{2t_d t_p\Delta^2} \\ n_1^d &= \frac{1}{2} + \frac{\vartheta\left(q_1 - q_2\right) + \left(w_1 - w_2\right)}{2t_d\Delta} \end{aligned} \tag{7}$$

Using the expressions of the market shares into (3), we can now express the generic hospital *i*'s profits as function of qualities and wages. Assumptions 1 and 2 ensure the existence of the equilibrium. Proposition 1 describes the equilibrium of the last stage of the game.

Assumption 1

Let us assume that k is sufficiently high, i.e. $k \geq \max\{k_1, k_2, k_3, k_4\}$, *where* $k_1 = \vartheta^2 / t_d \Delta$, $k_2 = (1+\vartheta)^2 / 4(t_d + t_p - \gamma)$, $k_3 = (1+\vartheta)(t_d + \gamma\vartheta) / 4 t_d t_p$, $k_4 = (t_d \Delta + \gamma\vartheta)^2 / 4\gamma\Delta^2 t_p t_d$.

Assumption 2

Let us assume that patient's mismatch costs are sufficiently high, i.e. $t_p > \max\left\{\left(-\gamma/\Delta + t_d/\vartheta + 2\sqrt{\left(t_d^2 + \left(\gamma\vartheta(t_d\Delta + \gamma\vartheta)\right)\Delta^2\right)/\vartheta^2}\right)/3, (t_d\Delta + \gamma\vartheta)^2 / 4\gamma\Delta\vartheta^2\right\}$.

Proposition 1

Assume that all agents are perfectly informed, that Assumptions 1 and 2 hold. Then hospitals' equilibrium quality and salary at stage three of the game are described by:

$$
\begin{aligned}
q &= \frac{p + t_p \Delta \vartheta}{2k t_p \Delta} \\
w &= \frac{p\gamma}{t_p \Delta} - t_d \Delta \\
\Pi &= \frac{1}{2}\left(p - \frac{p\gamma}{t_p\Delta} + t_d\Delta - \frac{\left(p + t_p\Delta\vartheta\right)^2}{4k t_p{}^2 \Delta^2} \right)
\end{aligned}
\tag{8}
$$

if $p > \bar{p} \equiv \dfrac{t_d t_p \Delta^2}{\gamma}$,

or by:

$$\begin{aligned} q &= \frac{p t_d \Delta + p\gamma\vartheta}{2k t_d t_p \Delta^2} \\ w &= 0 \\ \Pi &= \frac{p\left(4k_p^2 t_d^2 \Delta^4 - p\left(t_d \Delta + \gamma\vartheta\right)^2\right)}{8k t_p^2 t_d^2 \Delta^4} \end{aligned} \tag{9}$$

otherwise.

Proof

See Appendix A.

The equilibrium in the quality/salary subgame can be also described by the following comparative static results:

(a) if $p > \bar{p}$:

$$\begin{aligned} &\frac{dq}{dt_p} < 0 \quad \frac{dq}{dt_d} = 0 \quad \frac{dq}{d\gamma} = 0 \quad \frac{dq}{d\vartheta} > 0 \quad \frac{dq}{dp} > 0 \quad \frac{dq}{d\Delta} < 0 \quad \frac{dq}{dk} < 0 \\ &\frac{dw}{dt_p} < 0 \quad \frac{dw}{dt_d} < 0 \quad \frac{dw}{d\gamma} > 0 \quad \frac{dw}{d\vartheta} = 0 \quad \frac{dw}{dp} > 0 \quad \frac{dw}{d\Delta} < 0 \quad \frac{dw}{dk} = 0 \end{aligned} \tag{10}$$

(b) otherwise:

$$\frac{dq}{dt_p} < 0 \quad \frac{dq}{dt_d} < 0 \quad \frac{dq}{d\gamma} > 0 \quad \frac{dq}{d\vartheta} > 0 \quad \frac{dq}{dp} > 0 \quad \frac{dq}{d\Delta} < 0 \quad \frac{dq}{dk} < 0 \tag{11}$$

Results are intuitive. If $p > \bar{p}$, an increase in the distance between the hospitals, Δ, or in the mismatch costs lowers the degree of competition and in turn the incentive for hospitals to provide high quality or pay high salaries (note in addition that the assumption of indirect network externalities for the patients produces the result that an increase of patients' mismatch costs has a negative effect on salary: if competition for patients is softened, hospitals lose the incentive to compete fiercely for doctors and the salary decrease in equilibrium). If the regulator selects a higher price, the incentive to compete for patients increases and consequently hospitals tend to provide higher quality and pay doctors better. Clearly, an increase in the cost to provide quality has a negative effect on such provision, but not on salary selection. Quality (but not salary) is positively affected by an increase in ϑ, while the salary (but not the quality) is positively affect by an increase in γ (again, ceteris paribus, the more patients benefit from an increase of the number of doctors employed in a hospital, the higher the incentive for hospitals to pay higher salaries). In comparison to previous literature, now hospitals have one more reason to increase quality, e.g. to attract an higher number of doctors and, in turn, to be more competitive on the patients' side of the market. At the same time, the presence of network cross-group externalities explains why hospitals are willing to bear an extra cost (doctors' employment) in addition to investments in quality. If instead $0 < p \leq \overline{p}$, results are mostly unchanged regarding the equilibrium quality. However, now $dq / dt_d < 0$ and $dq / d\gamma > 0$. For such a range of prices hospitals wish to pay a negative salary to doctors, but we are not allowing negative salaries and in equilibrium salaries are equal to zero; it follows that if t_d decreases (the doctors' side of the market is more competitive) or γ increases (patients benefit more from interacting with doctors), hospitals react increasing quality (rather than salaries) to attract doctors.

We can now move to stage 2 of the game. Proposition 2 describes the location decisions of the two hospitals.

Proposition 2

For any given $p>0$ *and expecting pay-offs given either by* (8) *or* (9), *if Assumptions 1 and 3 hold, then both hospitals' location best response is to locate at the extremes of the unitary segment, i.e.* $\Delta=1$.

Proof

See Appendix

Proposition 2 contrasts to what has been shown in previous literature, in which hospitals might even decide to minimally differentiate. Now, adding a new side to the market, i.e. the doctors, increase the degree of competition to the point that it is always a best response to maximally differentiate from the competitor in order to soften competition.

In order to study stage 1 on the game, it is convenient to move to the analysis of the social optimum and, therefore, the price decision of the regulator. Let us describe, then, the welfare function the regulator aims to maximize. Given the assumptions of the model, the price paid by the patients and the wage received by the doctors do not create any distortion of the demand and supply of care. It follows that welfare is affected only by the location and quality decisions of the hospitals. Consequently, the welfare function is given by:

$$W=v^{p}+v^{d}+\frac{\gamma}{2}+q\left(1+\vartheta\right)-kq^{2}-\frac{t_{p}+t_{d}}{12}+\left(t_{p}+t_{d}\right)\frac{\Delta\left(1-\Delta\right)}{4} \qquad (12)$$

where the first four elements of the summation are respectively the benefit for patients and doctors to access a hospital, the positive externality for the patients when they interact with doctors, and the benefits for patients and doctors from accessing hospital quality; the fifth element is the quality enhancement costs that hospitals (symmetrically) incur; the last two terms are given by the mismatch costs patients and doctors bear to access the hospitals. If the regulator could directly choose quality and locations, the first best optimum would be described by:

$$\begin{aligned} q &= \frac{1+\vartheta}{2k} \\ \Delta &= \frac{1}{2} \end{aligned} \tag{13}$$

However, we are assuming that the regulator can not choose quality nor locations: he can at most affect quality through the price p, knowing that in the following stage hospitals would choose to maximally differentiate for any given price.

Proposition 3 describes the equilibrium of the game.

<u>Proposition 3</u>

If Assumptions 1 and 2 hold and $\gamma < t_d + t_p$*, in equilibrium the regulator chooses a price that generates the socially optimal quality provision, i.e.* $q = (1+\vartheta)/2k$ *and maximal product differentiation, i.e.* $\Delta = 1$ *. In particular, if* $t_d + t_p > \gamma > t_d$ *(and quality/salary competition is described by* (8)*), then the price will be equal to* $p^* = t_p$*; if* $\gamma \le t_d$ *(and quality/salary competition is instead described by* (9)*), then the price will be equal to* $p^{**} = (t_d t_p(1+\vartheta))/(t_d + \gamma\vartheta)$*. If, instead,* $\gamma \ge t_d + t_p$*, then the price will be equal to*

$p = 2kt_p\left(t_p-\gamma\right)+2t_p\sqrt{k\left(t_d+\left(t_p-\gamma\right)\left(kt_p-k\gamma-\vartheta\right)\right)}-t_p\vartheta>0$ *and the level of quality provided in equilibrium could be below or above the socially optimal one.*

Proof

See Appendix

From a social point of view, the model produces a clear result: the regulator is able to affect through price selection only the provision of quality; in particular, for values of γ sufficiently low, setting the price adequately the regulator can achieve the socially optimal quality provision. Introducing competition for doctors generates a fiercer degree of competition compared to the one described in previous contributions that justifies hospitals' location decision: no matter the price selected and for any combination of parameters, hospitals decide to maximally differentiate their services, as shown in proposition 2. When patients value relatively strongly the number of doctors employed in a hospital, i.e. high γ, the regulator can not indirectly regulate quality to its socially optimal value without the hospitals earning negative profits in equilibrium. The price that would ensure a socially optimal level of quality would be so high that hospitals would compete more fiercely for doctors to attract patients increasing salaries and quality (especially for high ϑ). It follows that for $\gamma \geq t_d + t_p$ the regulator can only choose the price that induces hospitals to earn profits equal to zero, with equilibrium quality excessive or insufficient from a social point of view.

Let us now briefly consider the case in which the regulator can commit to a price only after the two hospitals have chosen locations (partial commitment).

Corollary 1

Under partial commitment hospitals maximally differentiate their services and the equilibrium level of quality coincides with the socially optimal one if (a) $\gamma > t_d\Delta$ *and* $\hat{p}^* \leq \hat{p}_2$ *or (b)* $0 < \gamma \leq t_d\Delta$ *and* $\hat{p}^{**} \leq \hat{p}_2$*, where* $\hat{p}^* \equiv t_p\Delta$, $\hat{p}^{**} \equiv \dfrac{t_d t_p \Delta^2 (1+\vartheta)}{t_d\Delta + \gamma\vartheta}$ and

$$\hat{p}_2 \equiv t_p\Delta\left(2k\left(t_p\Delta - \gamma\right) - \vartheta\right) + 2t_p\sqrt{kt_p^2\Delta^2\left(t_d\Delta + \left(\gamma - t_p\Delta\right)\left(k\left(\gamma - t_p\Delta\right) + \vartheta\right)\right)} > 0 .$$

Otherwise, the regulator can at most select a price that ensures zero profits implying that specializations and quality can be excessive or insufficient compared to the social optimum.

Proof

See Appendix

3. Hospital competition with uninformed patients

This section studies the case in which patients have not complete information regarding the characteristics of their illness (i.e. the location on the segment) nor hospital characteristics. In particular, patients know only that they need hospital service and the distribution of the disease, say $Z = F(z)$. To obtain information patients can go to a general practitioner, GP, who instead is perfectly informed[3]. We assume that practitioners truthfully inform patients.

Let us suppose that a portion $\lambda \in (0,1]$ of patients visit a GP; consequently, a portion $(1-\lambda)$ of patients is not informed. Let us assume that such an uninformed portion of patients inelastically demands one unit of care with probability equal to 1/2 from either hospital. Following (K. Brekke et al., 2006), we assume cost heterogeneity with respect to

[3] In Brekke et al (2006) GPs' diagnosis is assumed not to be perfectly accurate.

GP consultations. Let $r \in [0,1]$ be the GP cost type of a patient and let *r* be uniformly distributed on the line. The cost to visit a GP is equal to *ar* where $a > 0$. The timing of the game with *strict gatekeeping* (i.e. $\lambda = 1$) as been described in section 1.

If instead we introduce the possibility that patients are free to choose whether to visit a GP (at a cost), i.e. *indirect gatekeeping*, we have to modify the first stage of the game not allowing the regulator to set λ, and introduce an additional stage just after quality and salary choice in order to let the patients to define endogenously λ. By the method of backward induction, let us first study the game under strict gatekeeping, analysing patients and doctors' hospital choice and hospitals' quality and salary competition, for given p, λ and specializations x_i, $i = 1,2$. For those patients who visit a practitioner and for doctors the utility is again given respectively by (1) and (2); the marginal agents indifferent to chose either hospitals are again located at $\bar{z}$ and $\bar{y}$, respectively given by (4) and (5). The demand of treatment for hospital one will be now given by:

$$\begin{aligned} n_1^p &= \lambda \bar{z} + \frac{(1-\lambda)}{2} \\ n_2^p &= 1 - n_1^p \end{aligned} \tag{14}$$

Hospitals share the market again as shown in (6).

Proposition 4 describes the equilibrium at the final stage of the game.

<u>Proposition 4</u>

Assume that a portion $\lambda \in (0,1]$ is informed and Assumptions 1 and 2 hold. Assume in addition that there exist maximum feasible levels of quality and salary given by $\bar{q}$ and $\bar{w}$.

Then the equilibrium level of salary and quality provided at the final stage of the game in equilibrium are given by:

$$q = \min\left\{\frac{t_p\Delta\vartheta + p\lambda}{2kt_p\Delta}, \overline{q}\right\}$$
$$w = \min\left\{\frac{p\gamma\lambda}{t_p\Delta} - t_d\Delta, \overline{w}\right\} \qquad (15)$$
$$\Pi = \frac{1}{2}\left(p + t_d\Delta - \frac{p\gamma\lambda}{t_p\Delta} - \frac{\left(t_p\Delta\vartheta + p\lambda\right)^2}{4kt_p{}^2\Delta^2}\right)$$

if $p > \frac{\overline{p}}{\lambda} = \frac{t_d t_p \Delta^2}{\gamma\lambda} \equiv \overline{\overline{p}}$ *, or*

$$q = \min\left\{\frac{p\left(t_d\Delta + \gamma\vartheta\right)\lambda}{2kt_d t_p\Delta^2}, \overline{q}\right\}$$
$$w = 0 \qquad (16)$$
$$\Pi = \frac{1}{2}\left(p - \frac{p^2\left(t_d\Delta + \gamma\vartheta\right)^2\lambda^2}{4kt_d{}^2 t_p{}^2\Delta^4}\right)$$

if otherwise.

Proof

See Appendix.

Easy comparative static exercises show that if $p > \overline{\overline{p}}$:

$$\frac{dq}{d\lambda} > 0 \quad \frac{dq}{dp} > 0 \quad \frac{dq}{dt_p} < 0 \quad \frac{dq}{dt_d} = 0 \quad \frac{dq}{d\gamma} = 0 \quad \frac{dq}{d\vartheta} > 0 \quad \frac{dq}{d\Delta} < 0 \quad \frac{dq}{dk} < 0$$
$$\frac{dw}{d\lambda} > 0 \quad \frac{dw}{dp} > 0 \quad \frac{dw}{dt_p} < 0 \quad \frac{dw}{dt_d} < 0 \quad \frac{dw}{d\gamma} > 0 \quad \frac{dw}{d\vartheta} = 0 \quad \frac{dw}{d\Delta} < 0 \quad \frac{dw}{dk} = 0 \qquad (17)$$

If $0 < p \leq \overline{\overline{p}}$:

$$\frac{dq}{d\lambda}>0 \quad \frac{dq}{dp}>0 \quad \frac{dq}{dt_p}<0 \quad \frac{dq}{dt_d}<0 \quad \frac{dq}{d\gamma}>0 \quad \frac{dq}{d\vartheta}>0 \quad \frac{dq}{d\Delta}<0 \quad \frac{dq}{dk}<0 \quad (18)$$

Changes in the parameters present qualitatively the same effects we have described for the case $\lambda=1$ in section two. Now, however, there is an additional parameter, λ, describing the degree of information of patients. When λ increases, clearly the degree of competition for patients increases as well, forcing hospitals to choose in equilibrium higher quality and salary (for $p>\bar{\bar{p}}$).

Given quality and salary described in proposition 4, hospitals in stage two choose their specialization. Proposition 5 describes hospitals' specialization decision when only a portion $\lambda>0$ of patients is informed.

Proposition 5

For any given $p>0$ *and* $0<\lambda\leq 1$, *expecting pay-offs given either by* (15) *or* (16), *holding Assumptions 1 and 2, both hospitals' location best response is to locate at the extremes of the unitary segment, i.e.* $\Delta=1$.

Proof

See Appendix

In contrast to what has been showed by (K. Brekke et al., 2006), hospitals have a clear and dominant incentive to maximally differentiate their service in order to soften competition. Since $0<\lambda\leq 1$, competition now could be milder than the one described in the previous section. However, since they have to compete for doctors as well as patients, hospitals always maximally differentiate.

We can now move to the first stage of the game in which a benevolent regulator sets the price of the treatment, p, and can impose a regime of strict gatekeeping, i.e. $\lambda = 1$, in order to maximize the welfare function. Given the assumption of the model, the welfare function is:

$$W = \frac{\gamma}{2} + q + \vartheta q - kq^2 - TC_d - TC_p - GP \tag{19}$$

where TC_d represents doctors' mismatch costs given by:

$$TC_d = \frac{t_d}{12} - \frac{t_d \Delta(1-\Delta)}{4}. \tag{20}$$

TC_p represents instead patients' mismatch costs and it is given by the weighted sum of the mismatch costs of informed and uninformed patients:

$$TC_p = (1-\lambda)\left(\frac{t_p}{12}\left(1+3\Delta^2\right)\right) + \lambda\left(\frac{t_p}{12} - \frac{t_p}{4}\Delta(1-\Delta)\right) \tag{21}$$

Finally, GP represents the cost incurred by the λ portion of patients who decided to visit a GP:

$$GP = \frac{a\lambda^2}{2} \tag{22}$$

The regulator's objective function is therefore given by:

$$W = \frac{\gamma}{2} + (1+\vartheta)q - kq^2 + \frac{t_p}{12}\left(3\Delta(\lambda-\Delta)-1\right) - \frac{1}{2}a\lambda^2 - \frac{t_d}{12}\left(1+3\Delta(\Delta-1)\right) \tag{23}$$

As pointed out already by (K. Brekke et al., 2006) assuming that hospitals do not compete for doctors, the λ that maximizes welfare is identical to the one that maximizes the patients' benefit to visit a GP net of the costs, i.e. social and private incentives to GP attendance coincide and $\lambda^* = \frac{t_p\Delta}{4a}$. Indeed, if patients were free to decide whether to visit a GP or the regulator were able to set any $\lambda \in (0,1]$, for a given price, the equilibrium of the game would be given therefore by:

$$\begin{aligned}
&\lambda^*(p) = \frac{t_p}{4a} \\
&\Delta^* = 1 \\
&q^*(p) = \frac{p + 4a\vartheta}{8ka} \\
&w^*(p) = \frac{p\gamma}{4a} - t_d
\end{aligned} \tag{24}$$

if $p > \overline{\overline{p}}$, or by:

$$\begin{aligned}
&\lambda^*(p) = \frac{t_p}{4a} \\
&\Delta^* = 1 \\
&q^*(p) = \frac{p(t_d + \gamma\vartheta)}{8kat_d} \\
&w^*(p) = 0
\end{aligned} \tag{25}$$

otherwise.

Proposition 6 describes the comparative static properties of the specialization-quality-consultation equilibrium. It could be directly compared to Proposition 2 in (K. Brekke et al., 2006).

Proposition 6

The specialization-quality-consultation equilibrium presents the following comparative static properties:

(i) if $p > \bar{\bar{p}}$ *, GP attendance is increasing in patients' mismatch costs and decreasing in GP consultation costs; quality is increasing in the treatment price and the benefit that doctors receive from hospital quality,* ϑ *, and decreasing in the quality cost k and GP consultation costs;*

(ii) if $0 < p \leq \bar{\bar{p}}$ *, GP attendance is still described as in (i). However, an additional negative effect is generated to equilibrium quality provision if the doctors' mismatch costs increases;*

(iii) salary is increasing in treatment price, he benefit that doctors receive from hospital quality, ϑ *, and decreasing in the quality cost k, GP consultation costs and doctors' mismatch costs.*

While the description of equilibrium salary is quite intuitive, there are some clear differences between the results described in proposition 5 and those reported in proposition 2 in (K. Brekke et al., 2006) and all of them are generated by the maximal differentiation result described in proposition 4 above. For example, in (K. Brekke et al., 2006) an increase in *k*, would decrease equilibrium quality provision and consequently hospitals' incentive to differentiate; in turn, a decrease in Δ would decrease the benefits of consulting a GP, having a negative effect on λ. With a similar reasoning (K. Brekke et al., 2006) explain the positive relationship between λ and the price or patients' mismatch costs. However, since in our model $\Delta = 1$ regardless the regulated price and GP attendance, in equilibrium λ is not affected by patients' mismatch costs, quality costs nor treatment price. Regarding equilibrium quality provision, our results are in line with the literature. In addition, there is one more parameter that affect quality provision: an increase in ϑ has a direct positive effect on quality provision. When *p* is sufficiently small, also doctors' mismatch costs have a (negative) effect on quality: if doctors' mismatch costs decreases competition for doctors is fiercer and hospitals have forced to provided higher quality.

For a given price, quality expressed in (24) and (25) are not necessarily optimal and if the regulator can at most impose a strict gatekeeping system, i.e. $\lambda = 1$, we have to consider when, for quality given by (15) or (16) and for $\Delta^* = 1$, the first derivative of the welfare function with respect to λ is non negative, imposing $\lambda = 1$. Such a derivative is given by:

$$\left.\frac{\partial W}{\partial \lambda}\right|_{\lambda=1} = \frac{2pt_p - 2p^2 + kt_p^2\left(t_p - 4a\right)}{4kt_p^2} \tag{26}$$

if $p > \overline{\overline{p}}$, or

$$\left.\frac{\partial W}{\partial \lambda}\right|_{\lambda=1} = \frac{kt_p^2 t_d^2\left(t_p - 4a\right) + 2pt_p t_d\left(1+\vartheta\right)\left(t_d + \gamma\vartheta\right) - 2p^2\left(t_d + \gamma\vartheta\right)^2}{4kt_p^2 t_d^2} \tag{27}$$

otherwise.

Whether $\lambda = 1$ is a socially optimal strategy depends on the parameters of the model (price for the moment is assumed to be exogenous). In particular, for a price p and GP consulting cost a sufficiently low and for doctors' mismatch costs t_d .

Let us now introduce the possibility that the regulator can set the price p to maximize W . Given the fact that the private incentive to visit a GP is equal to the socially optimal one, the regulator has only to set p and let patients free to decide whether to visit a GP. In other words, as already argued by (K. Brekke et al., 2006), it is not necessary to set up a strict gatekeeping system if the regulator can set the treatment price in order to maximize welfare. Proposition 7 describes the equilibrium of the game.

<u>Proposition 7</u>

If Assumption 1 holds, $\gamma > t_d$ *and a sufficiently large, i.e.* $a \geq \left(4k\left(\gamma - t_d\right) + \left(1+\vartheta\right)^2\right) \Big/ 16k$ *, in equilibrium the regulator chooses a price,* $p = p^* = 4a$ *that ensures the optimal provision of quality. If instead* $0 < a < \left(4k\left(\gamma - t_d\right) + \left(1+\vartheta\right)^2\right) \Big/ 16k$ *the regulator, to ensures that hospitals do not earn negative profits, choose the price:* $p = 8ka\left(4a - \gamma\right) - 4a\vartheta + 8a\sqrt{k\left(t_d + \left(4a-\gamma\right)\left(4ka - k\gamma - \vartheta\right)\right)} > 0$ *and quality provision can excessive or insufficient from a social point of view.*

If $0 < \gamma \leq t_d$ *and a is sufficiently high, i.e.* $a \geq \left(1+\vartheta\right)\left(t_d + \gamma\vartheta\right)/16kt_d^2$*, in equilibrium the regulator again chooses a price,* $p = p^{**} = 4at_d\left(1+\vartheta\right)/\left(t_d + \gamma\vartheta\right)$ *that ensures the optimal provision of quality. If instead* $0 < a < \left(1+\vartheta\right)\left(t_d + \gamma\vartheta\right)/16kt_d^2$ *the regulator, to ensures that hospitals do not earn negative profits, choose the price:* $p = 64kt_d^2a^2/\left(t_d + \gamma\vartheta\right)^2 > 0$ *and quality provision can excessive or insufficient from a social point of view. For any combination of parameters the equilibrium level of horizontal differentiation is maximal and therefore excessive compared to the social optimum.*

Proof

See Appendix.

Results described in the previous section hold also for this version of the model with imperfect information. The regulator can choose one variable , the price, and consequently can at most affect the provision of quality to the socially optimal level. In contrast with previous literature, the level of horizontal differentiation in the hospital market will always be maximal, since now hospitals compete simultaneously for patients (through quality and number of doctors employed) and for doctors (through salaries).

4. Conclusions

The paper studied the competition between two identical hospitals in a two-sided market composed on one side by patients who require hospital treatment and on the other side by doctors who obtain positive utility from working in a hospital. Assuming that both patients and doctors present a continuum of preferences with respect to the disease space (the unit segment), hospitals compete simultaneously for both groups of agents selecting the specialization first, the quality of the service and the salary for the doctors then. The price of the hospital treatment is centrally set by a benevolent regulator (a similar system can correspond to the DRG-pricing system), and, if patients are not informed, a strict gatekeeping system can be set. In contrast to what has been demonstrated in relevant previous literature, the paper shows that for any positive price selected by the regulator and for any level of information among patients, hospitals maximally differentiate their services. Introducing salary competition for doctors increases the level of competition between hospitals and in turn justifies a centrifugal force in the market that generates a level of product differentiation always excessive compared to the social optimum. The regulator, however, can still indirectly affect the quality provision that, when patients experience sufficiently low cross-group externalities with respect to doctors employed in a hospital, in equilibrium coincides with the socially optimal one.

Appendix

Proof of Proposition 1

Substituting expressions (7) into (3), the profits for hospital i , $i=1,2$, $j=1,2\neq i$:

$$\Pi_i=\frac{p\left(t_d\Delta\left(q_i-q_j+t_p\Delta\right)+\gamma\left(w_i-w_j+\vartheta\left(q_i-q_j\right)\right)\right)-t_p\Delta\left(kq_i^2t_d\Delta+w_i\left(w_i-w_j+t_d\Delta+\vartheta\left(q_i-q_j\right)\right)\right)}{2t_dt_p\Delta^2}\quad(28)$$

Second order conditions, SOCs, for profit maximization are given by $\partial^2\Pi_i/\partial q_i^2=-k$, $\partial^2\Pi_i/\partial w_i^2=-1/t_d\Delta$, $Det(H)=-\left(-4kt_d\Delta+\vartheta^2\right)/\left(4t_d^2\Delta^2\right)$, where H is the Hessian matrix of the profit function, and are satisfied if $4kt_d\Delta>\vartheta^2$, that is true given the Assumption 1.

First order conditions, Focs, are given by:

$$\begin{aligned}\frac{\partial\Pi_i}{\partial q_i}&=\frac{p\left(t_d\Delta+\gamma\vartheta\right)-t_p\Delta\left(2kq_it_d\Delta+w_i\vartheta\right)}{2t_dt_p\Delta^2}=0\\ \frac{\partial\Pi_i}{\partial w_i}&=\frac{p\gamma+t_p\Delta\left(w_j-2w_i-t_d\Delta-q_i\vartheta+q_j\vartheta\right)}{2t_dt_p\Delta^2}=0\end{aligned}\quad(29)$$

Solving simultaneously the Focs, the candidate subgame symmetric (i.e. $q_1=q_2=q$ and $w_1=w_2=w$) equilibrium is described by (8). The salary in (8), for a range of prices sufficiently low, i.e. $0<p\leq\overline{p}$, could be negative. In such a case, since we are imposing that hospitals can not select a negative salary, hospitals set a salary equal to zero and the candidate equilibrium in (9) follows. To prove that the quality and the wage in (8) and (9) are indeed an equilibrium, we have to prove that no hospital has the incentive to undercut the competitor on either side of the market. Let us consider first the patients' side of the market. To treat all patients, either of the two hospitals, say hospital 1, has to choose a combination of quality and wages such that even the patient located at the opposite extreme of the segment is willing to be treated by the undercutting deviating hospital. Hospital 1 has therefore to choose:

$$U_1^p(1-x_1) \geq U_2^p(1-x_2) \quad \Rightarrow \quad q_1 \geq q_2 + t_p\Delta + \gamma\left(1-2n_1^d\right) \tag{30}$$

where n_1^d is given by (7). The deviating hospital has no incentive to choose a quality higher than the one expressed in (30). Suppose $p > \bar{p}$, substituting the expression of quality given by (30) into (7) and (3), and imposing that the quality and salary chosen by hospital 2 are given by (8) first, we obtain hospital 1's profits as a function of the wage w_1. Such profits are maximized for

$$w_1 = \frac{2k\gamma\left(p\gamma^2 + t_d t_p \Delta^2\left(t_p\Delta - \gamma\right)\right) - \left(t_d\Delta + \gamma\vartheta\right)\left(t_p\Delta^2\left(2t_d + t_p\vartheta\right) - 2p\gamma\right)}{2t_p\Delta\left(t_d\Delta + \gamma\left(k\gamma + \vartheta\right)\right)} \tag{31}$$

Substituting the wage given by (31) into (30), we have that the required quality to undercut is given by:

$$q_1 = \frac{1}{2}\left(\frac{p}{kt_p\Delta} + t_p\Delta + \frac{\vartheta}{k} + \frac{t_p\Delta\left(t_d\Delta - k\gamma^2\right)}{t_d\Delta + \gamma\left(k\gamma + \vartheta\right)}\right) \tag{32}$$

The profits of deviation for hospital 1 are therefore given by:

$$\hat{\Pi}_1 = \frac{1}{8k}\left[\frac{\left(t_d\Delta + kt_p{}^2\Delta^2\left(\gamma^2 - t_p{}^2\Delta^2\right) + \gamma\vartheta\right)\left(4kt_d\Delta - \vartheta^2\right)}{t_d\Delta + \gamma\left(k\gamma + \vartheta\right)} - \frac{p^2}{t_p{}^2\Delta^2} - \frac{2p\left(2k\left(\gamma - t_p\Delta\right) + \vartheta\right)}{t_p\Delta}\right] \tag{33}$$

and they are less than the profits given by (8). Let us now consider the case $0 < p \leq \bar{p}$ and substitute the expression of quality given by (30) into (7) and (3), and impose that the quality and salary chosen by hospital 2 are given by (9). Following the previous reasoning, we can show that the profits of deviation are less than the profits given by (9) as long as $k \geq k_1$ and $t_p > \left(-\gamma/\Delta + t_d/\vartheta + 2\sqrt{\left(t_d^2 + \left(\gamma\vartheta\left(t_d\Delta + \gamma\vartheta\right)\right)\Delta^2\right)/\vartheta^2}\right)/3 > 0$, as assumed in Assumptions 1 and 2. Since hospital 2 is perfectly symmetric, no hospital has incentive to undercut the competitor on the patients' side of the market. We have now to prove that the same is true for the doctors' side too. In order to attract all doctors in the market the deviating hospital, say again hospital 1, has to offer a combination of salary and quality such that:

$$U_1^d\left(1-x_1\right) \geq U_2^d\left(1-x_2\right) \quad \Rightarrow \quad w_1 \geq \vartheta q_2 + w_2 + t_d\Delta - \vartheta q_1 \tag{34}$$

Hospital 1 clearly has no incentive to choose a salary higher than the one expressed in (34) with equality. If $p > \overline{p}$ and we substitute the salary given by (34) into (7) and (3), and imposing that the quality and salary chosen by hospital 2 are given by (8), we obtain the profits of the deviating hospital as a function of its quality, i.e. q_1 . The quality that maximizes the deviating hospital's profits is:

$$q_1 = \frac{p + 2t_p\Delta\vartheta}{2kt_p\Delta} \tag{35}$$

and the undercutting salary follows from (34):

$$w_1 = \frac{p\gamma}{t_p\Delta} - \frac{\vartheta^2}{2k} \tag{36}$$

Assumption 1 ensures that the salary given in (36) is positive and profits generated from deviation are less than profits given by (8). Similarly, if $0 < p \leq \overline{p}$ and hospital 1 deviates from equilibrium described in (9), it can not obtain higher profits. Hospital 2's behaviour is perfectly symmetrical and that proves that no hospital has incentive to undercut the competitor. Finally, in order to prove that the expressions in (8) and (9) represent indeed an equilibrium under the assumption of complete information we need to prove that no hospital has the incentive to leave the market for doctors and to attract patients through quality provision. In order to leave the market for doctors, the deviating hospital, say again hospital 1, has to choose a salary such that:

$$U_1^d(x_1) \leq U_2^d(x_2) \quad \Rightarrow \quad w_1 < \vartheta q_2 + w_2 - t_d \Delta - \vartheta q_1 \tag{37}$$

If $p < \overline{p}$, deviating from equilibrium described by (8), for $p < t_p \Delta\left(4kt_d \Delta - \vartheta^2\right)/(2k\gamma + \vartheta)$ even for a deviating quality $q_1 = 0$, hospital 1 should choose a negative to leave the doctors' side of the market, but given that we are not allowing such a possibility in the model, that is not feasible. For $t_p\left(4kt_d \Delta^2 - \Delta\vartheta^2\right)/2k\gamma > p > t_p \Delta\left(4kt_d \Delta - \vartheta^2\right)/\left(2k\gamma + \vartheta\right)$, there is a non empty subset of quality and salary for which the deviating hospital can not attract any doctor; however, for such a range of p the quality that would maximize the profits of deviation would again involve a negative salary. Therefore, for in such a case hospital 1 can at most choose $w_1 = 0$ and the corresponding quality that would ensure not to employ any doctor would be given by $q_1 = \left(2kp\gamma - 4kt_d t_p \Delta^2 + p\vartheta + t_p \Delta\vartheta^2\right)/2kt_p\Delta\vartheta$. It can be shown that the profits that follow from such kind of deviation are less than those reported in (8) as long as $k > k_1$. The same is also true for higher if $p > t_p\left(4kt_d \Delta^2 - \Delta\vartheta^2\right)/2k\gamma$, that involves a positive salary of deviation. If $0 < p \leq \overline{p}$ and hospital 1 is deviating from the equilibrium described by (9) (according to the deviation condition (37)), the salary that would ensure profits maximization in deviation would be negative given that $k > k_1$, requiring the hospital to choose a salary equal to zero. It follows that the profits of deviation are again less than profits when salary and quality are given by

(9). Symmetry ensures that hospital 2 is not willing to deviate either, concluding this proof. Q.E.D.

Proof of Proposition 2

Given pay-offs in (8) or (9), Socs are easily verified. The first derivative of profits with respect to locations is given respectively by:

$$\frac{\partial \Pi}{\partial x_2} = -\frac{\partial \Pi}{\partial x_1} = \frac{p^2 + 2kt_d t_p{}^2\Delta^3 + pt_p\Delta(2k\gamma + \vartheta)}{4kt_p{}^2\Delta^3} > 0 \tag{38}$$

if profits are given by (8), or by:

$$\frac{\partial \Pi}{\partial x_2} = -\frac{\partial \Pi}{\partial x_1} = \frac{p^2(t_d\Delta + 2\gamma\vartheta)(t_d\Delta + \gamma\vartheta)}{4kt_d{}^2t_p{}^2\Delta^5} > 0 \tag{39}$$

It follows that the two hospitals locate at the extremes of the segment at stage two, i.e. $\Delta = 1$. Q.E.D.

Proof of Proposition 3

Suppose first that $t_p + t_d > \gamma > t_d$. Substituting the expression for quality given by (8) into the welfare function (12), imposing $\Delta = 1$ and maximizing[4] with respect to p, we obtain $p^* = t_p > \overline{p}$. The price that maximize the welfare function (12) when quality is given by (9) would be given by $p^{**} = (t_d t_p(1+\vartheta))/(t_d + \gamma\vartheta) > \overline{p}$. It follows that for $t_p + t_d > \gamma > t_d$ the best feasible price the regulator can choose to ensure that the equilibrium quality and salary are given by (9) is $\overline{p}$. Since $W(\overline{p}) < W(p^*)$, it follows that for $t_p + t_d > \gamma > t_d$ the

[4] Socs are always satisfied.

candidate optimum price is p^*. Profits are concave in p and equal to zero for $p_{1,2} = 2kt_p\left(t_p-\gamma\right) \pm 2t_p\sqrt{k\left(t_d+\left(t_p-\gamma\right)\left(kt_p-k\gamma-\vartheta\right)\right)} - t_p\vartheta > 0$, $p_2 \geq p_1$. If $k \geq k_2$, as described in Assumption 1, then $p_1 \geq p^* \geq p_2$, and profits are greater than zero in equilibrium. For extreme values of γ, $p = p^*$ can not be an equilibrium. If $\gamma > t_p + t_d$, $p^* > p_2$, implying negative profits in equilibrium. Therefore the regulator can at most choose $p = p_2$ with equilibrium profits equal to zero. Finally, for $0 < \gamma < t_d$, we have that $p^* < p^{**} < \overline{p}$, implying that the best feasible price for the regulator to impose the regime described in (8) is given by $\overline{p}$. Since $W\left(\overline{p}\right) < W\left(p^{**}\right)$, it follows that for $t_d > \gamma > 0$ the candidate optimum price is p^{**}. Profits are again concave (now since p^{**} would induce an equilibrium with salaries equal to zero, profits are equal to zero for $p = \left\{0, (4kt_d^2t_p^2/(t_d+t_p)^2)\right\}$). p^{**} ensures positive profits in equilibrium for $k \geq k_3$ as defined in Assumption 1. Q.E.D

Proof of Corollary 1

Stage 3 competition produces again the expression (8) and (9) in Proposition 1.

In stage 2 the regulator selects the price to maximize the welfare function. The welfare function is given by substituting into (12) the quality expressed (8) if $p > \overline{p}$, and the quality expressed in (9) if $0 < p \leq \overline{p}$. If $\gamma > t_d\Delta$, the welfare function is maximized (and stage 3 continuation produces the socially optimal level of quality) for $p = \hat{p}^* \equiv t_p\Delta$. In stage 1 profits are strictly concave in $p > 0$ and equal to zero for

$p = \hat{p}_{1,2} \equiv t_p \Delta \left(2k \left(t_p \Delta - \gamma \right) - \vartheta \right) \pm 2 t_p \sqrt{k t_p^2 \Delta^2 \left(t_d \Delta + \left(\gamma - t_p \Delta \right) \left(k \left(\gamma - t_p \Delta \right) + \vartheta \right) \right)} > 0$. It follows that the best strategy for the regulator when $\gamma > t_d \Delta$, is to choose $p = \min\left\{ \hat{p}^*, \hat{p}_2 \right\}$.

In stage 1 hospitals, if expecting $p = \hat{p}^*$, would choose to maximally specialize their services; in fact, the two hospitals have to choose specializations to maximize profits:

$$\Pi = \frac{4k\left(\Delta\left(t_d + t_p\right) - \gamma\right) - \left(1 + \vartheta\right)^2}{8k} \tag{40}$$

obtaining $\frac{\partial \Pi}{\partial \Delta} = \frac{t_p + t_d}{2}$ and $\Delta = 1$. Otherwise, expecting zero profits in stage 3, the specialization choice does not matter anymore: there is a continuum of equilibria, depending on $\Delta \in [0,1]$, where the equilibrium quality can be excessive or insufficient from a social point of view.

If instead $0 < \gamma \le t_d$, the welfare function is maximized (and stage 3 continuation produces the socially optimal level of quality) for $p = \hat{p}^{**} \equiv \frac{t_d t_p \Delta^2 \left(1 + \vartheta\right)}{t_d \Delta + \gamma\vartheta}$. It follows that the best strategy for the regulator when $\gamma > t_d \Delta$, is to choose $p = \min\left\{ \hat{p}^{**}, \hat{p}_2 \right\}$

In stage 1 hospitals, if expecting $p = \hat{p}^{**}$, would choose to maximally specialize their services; in fact, the two hospitals have to choose specializations to maximize profits:

$$\Pi = \frac{\left(1 + \vartheta\right)\left(t_d \Delta \left(4k t_p \Delta - \vartheta - 1\right) - \left(1 + \vartheta\right) \gamma\vartheta\right)}{8k\left(t_d \Delta + \gamma\vartheta\right)} \tag{41}$$

and it follows that $\frac{\partial \Pi}{\partial \Delta} = \frac{t_p + t_d}{2}$, implying that the two hospitals will locate at the extremes of the segment again. If instead $\hat{p}^{**} < \hat{p}_2$, expecting zero profits in stage 3, there is a continuum of equilibria, depending on $\Delta \in [0,1]$, where the equilibrium quality can be excessive or insufficient from a social point of view. Q.E.D.

Proof of Proposition 4

Given that $\lambda \in (0,1]$ and given Assumptions 1 and 2 it can be easily shown (following the same procedure we used for proof of Proposition 1) that no hospital has the incentive to deviate from the equilibria described in Proposition 4 undercutting the competitor on either side of the market, nor leaving the doctors' side of the market choosing a salary sufficiently low. However, given the possibility that a portion $(1-\lambda)$ of patients is not informed, another possibility of deviation arises: hospitals might decide to serve only the uniformed portion of patients and pay a salary equal to zero and provide quality level equal to zero. If a hospital decides to do so, it will earn profits equal to:

$$\hat{\Pi} = p\frac{(1-\lambda)}{2} \tag{42}$$

Let us first suppose that $p > \overline{\overline{p}}$ and define the difference between the profits given by (15) and those given by (42) as:

$$\phi(p,\lambda) = \Pi - \hat{\Pi} = \frac{4kt_p\Delta\left(t_p\Delta\left(t_d\Delta + p\lambda\right) - p\gamma\lambda\right) - \left(t_p\Delta\vartheta + p\lambda\right)}{8kt_p^2\Delta^2} \tag{43}$$

It can be easily verified that $\partial^2\phi(p,\lambda)/\partial p^2 < 0$, $\phi(0,\lambda) > 0$, $\lim_{p\to\infty}\phi(p,\lambda) = -\infty$. So there is a limit price, say $\hat{p} = \left\{p \in (0,\infty)\ s.t.\ \phi(p,1) = 0\right\}$, such that for $\overline{\overline{p}} < p \le \hat{p}$ then

$\phi(p,1) \geq 0$. Given the monotonic relationship between *q* and *w* with respect to *p* and λ, a unique salary and quality level will correspond to such limit price $\hat{p}$. In particular, $\hat{p} = t_p\Delta\left(2kt_p\Delta - \vartheta - 2k\gamma\right) + 2\sqrt{kt_p^2\Delta^2\left(t_d\Delta + \left(\gamma - t_p\Delta\right)\left(k\left(\gamma - t_p\Delta\right) + \vartheta\right)\right)}$ (well defined and larger than $\overline{\overline{p}}$ if Assumptions 1 and 2 hold). Let us define $\tilde{p} \equiv \min\left\{\hat{p} \middle| \Delta \in [0,1]\right\}$ and $\overline{q} \equiv \arg\max\left[\Pi\left(p = \tilde{p}, \lambda = 1\right)\right]$ and $\overline{w} \equiv \arg\max\left[\Pi\left(p = \tilde{p}, \lambda = 1\right)\right]$(given the assumption of the model, the corner solutions are symmetric). The upper bounds on quality and wages ensure that for prices higher than $\hat{p}$, hospitals have no incentive to deviate and serve only uninformed patients. It can be easily verified that deviation from the corner solution (in which hospitals select $q_1 = q_2 = \overline{q}$ and $w_1 = w_2 = \overline{w}$) is not profitable.

If instead, $0 < p \leq \overline{\overline{p}}$ the candidate equilibrium is given by (16). The deviation profits are again given by (42) and the difference with profits given by (16) is now given by:

$$\hat{\phi}(p,\lambda) = \frac{1}{8}p\lambda\left(4 - \frac{p\lambda\left(t_d\Delta + \gamma\vartheta\right)^2}{kt_d^2t_p^2\Delta^4}\right) \tag{44}$$

It can be easily verified again that $\partial^2\hat{\phi}(p,\lambda)/\partial p^2 < 0$, $\hat{\phi}(0,\lambda) = 0$, $\lim_{p\to\infty}\hat{\phi}(p,\lambda) = -\infty$. So there is again a limit price, say $\hat{\hat{p}} = \left\{p \in (0,\infty)\ s.t.\ \hat{\phi}(p,1) = 0\right\}$, such that for $p \leq \hat{\hat{p}}$ then $\hat{\phi}(p,1) \geq 0$. Since, $\hat{\hat{p}} = \left(4kt_d^2t_p^2\Delta^4\right)/\left(t_d\Delta + \gamma\vartheta\right)^2 > \overline{\overline{p}}$ it follows that for $0 < p \leq \overline{\overline{p}}$ hospitals have no incentive deviate from the equilibrium and serve only uninformed patients.

Proof of Proposition 5

For any give price and λ hospitals choose specializations x_i, $i = 1,2$ to maximize profits given by (15) or (16).

The Socs are always satisfied and the Focs are given by:

$$\frac{\partial \Pi}{\partial x_1} = -\frac{\partial \Pi}{\partial x_2} = -\frac{2kt_d t_p^2 \Delta^3 + pt_p \lambda \Delta \left(2k\gamma + \vartheta\right) + p^2 \lambda^2}{4kt_p^2 \Delta^3} < 0 \tag{45}$$

if $p > \overline{\overline{p}}$, or

$$\frac{\partial \Pi}{\partial x_1} = -\frac{\partial \Pi}{\partial x_2} = -\frac{p^2 \lambda^2 \left(t_d \Delta + 2\gamma\vartheta\right)\left(t_d \Delta + \gamma\vartheta\right)}{4kt_p^2 \Delta^3} < 0 \tag{46}$$

if $0 < p \leq \overline{\overline{p}}$. It follows that hospitals always maximally differentiate, regardless the price chosen by the regulator or the number of patients that decide to the visit a GP. Q.E.D.

Proof of Proposition 7

Let us suppose first that $\gamma > t_d$. Substituting the equilibrium values expressed in (24) into (23), we obtain the expression of welfare as a function (strictly concave) of price, maximized (implying optimal quality provision) for $p = p^* = 4a$, when in the final stage of the game doctors receive a positive salary. If instead we substitute the equilibrium values expressed in (25) into (23), we obtain the expression of welfare as a function (strictly concave) of price, maximized (implying optimal quality provision) for $p = p^{**} = 4at_d\left(1+\vartheta\right)/\left(t_d + \gamma\vartheta\right)$, when in the final stage of the game doctors receive salary equal to zero. Given that $\gamma > t_d$, p^{**} is not feasible, since $p^{**} > \overline{\overline{p}}$ and p^* is the candidate equilibrium price. If $0 < a < \left(4k\left(\gamma - t_d\right) + \left(1+\vartheta\right)^2\right) / 16k$, $\Pi\left(p^*\right) < 0$ and the regulator has to choose the price that at least ensure non negative hospital profits (not optimal quality provision): $p = 8ka\left(4a-\gamma\right) - 4a\vartheta + 8a\sqrt{k\left(t_d + \left(4a-\gamma\right)\left(4ka - k\gamma - \vartheta\right)\right)} > 0$.

If $0<\gamma\leq t_d$, p^* is not feasible, since $p^*<\overline{\overline{p}}$ and p^{**} is the candidate equilibrium price. If $0<a<(1+\vartheta)(t_d+\gamma\vartheta)/16kt_d^2$, $0<a<\left(4k(\gamma-t_d)+(1+\vartheta)^2\right)\big/16k$, $\Pi\left(p^{**}\right)<0$ and the regulator has to choose the price that at least ensure non negative hospital profits (not optimal quality provision): $p=64kt_d^2a^2/(t_d+\gamma\vartheta)^2>0$. The regulator can ensure optimal quality provision choosing prices p^* or p^{**} respectively if $\gamma>t_d$ and $a\geq\left(4k(\gamma-t_d)+(1+\vartheta)^2\right)\big/16k$ or if $0<\gamma\leq t_d$ and $a\geq(1+\vartheta)(t_d+\gamma\vartheta)/16kt_d^2$.Q.E.D.

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